資工系物理

大學的物理強調的是定義、定律的了解、應用，以及加強題目分析的能力，和高中強調計算有所不同，所以學習上請多花時間去了解物理量的定義及物理定律的涵義。
資工系的物理為一學期，主要的課程內容為電學，需要較強的數學基礎，如向量、微分、積分概念以及簡單的微分方程，所以請利用時間多複習及預習。以下為一些基礎講義。

[第一單元] 數學基礎:

A. 向量:

A-1

- The vector equation is equivalent to three equations:

$\vec{R}=\vec{A}+\vec{B}$

$R_x = A_x + B_x \quad R_y = A_y + B_y \quad R_z = A_z + B_z$

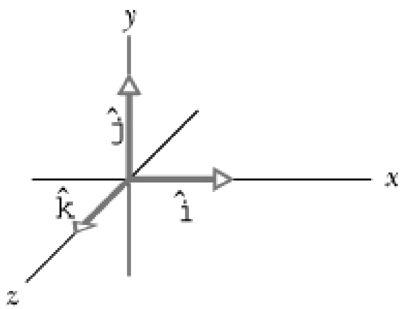
- The scalar (dot) product of two vectors is

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

- The vector (cross) product of two vectors is

$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$$

where the direction of **n** is given by the right-hand rule.

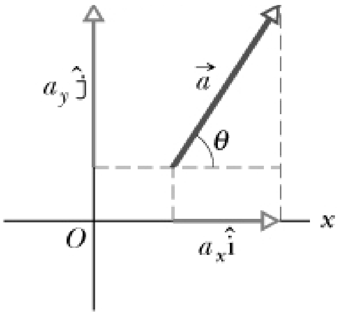


A-2 參考座標系

二維座標:(1)笛卡兒座標$(x,\ y)$(2)極座標 (r,θ)

三維座標:(1)笛卡兒座標(x,y,z)

(2)圓柱座標(r,θ,z) (3)球座標(r,θ,φ)

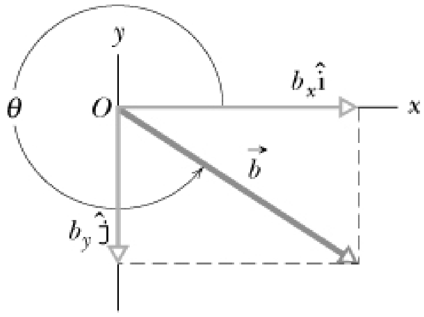


(a)

- 單位向量(unit vector)

$(\hat{i},\hat{j},\hat{k})$; $(\hat{r},\hat{\theta},\hat{z})$

$\vec{a}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$; $\vec{b}=b_x\hat{i}+b_y\hat{j}+b_z\hat{k}$



(b)

Products of Vectors

Let θ be the smaller of the two angles between $\vec{a}$ and $\vec{b}$. Then

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = a_x b_x + a_y b_y + a_z b_z = ab \cos \theta$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

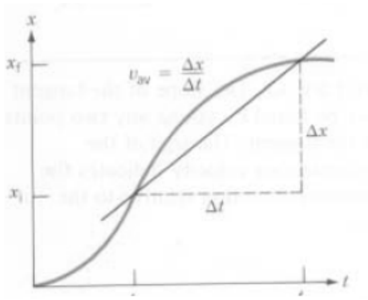
$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

B. 微積分

$$\frac{d}{dx}(ax^n) = anx^{n-1} \quad ; \int ax^n dx = \frac{a}{n+1} x^{n+1} \quad a\text{:常數}$$

Derivatives and Integrals

1.微分: 斜率 (以速度-位移 與 功爲例)



$$\frac{d}{dx} \sin x = \cos x \qquad \int \sin x \, dx = -\cos x$$

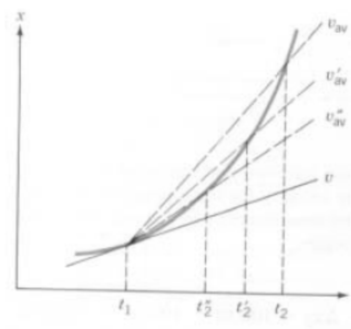
$$\frac{d}{dx} \cos x = -\sin x \qquad \int \cos x \, dx = \sin x$$

$$\frac{d}{dx} e^x = e^x \qquad \int e^x \, dx = e^x$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

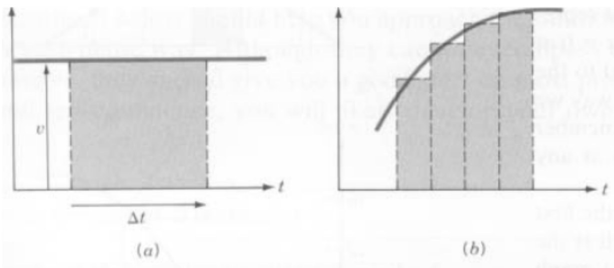
$$\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$$

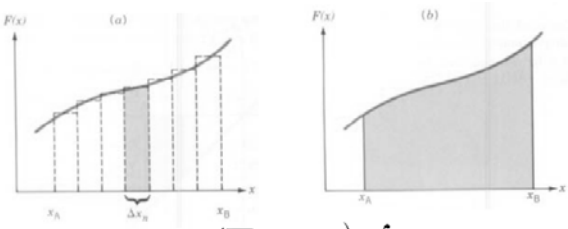
$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$



2.積分 ： 面積-------以 速度-位移 與 功爲例

(A) 速度 velocity --- 位移 displacement *(B)功 work*





$$W = \lim_{\Delta x_n \to 0} \left(\sum F_n \cdot \Delta x_n \right) = \int F_x \, dx$$

Thus, the work done by a force F_x from an initional point A to final point B is

$$W_{A \to B} = \int_{XA}^{XB} F_x \, dx$$

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3.微積分: 以質心爲例

To find the center of mass of a continuous body one must integrate the contributions of each mass element *dm*.

$$\mathbf{r}_{cm} = \frac{1}{M} \int_{body} \mathbf{r} dm = \frac{1}{M} \int_{body} \mathbf{r} \, \lambda(\mathbf{r}) \, dr$$

$$= \frac{1}{M} \int_{body} \mathbf{r} \, \sigma(\mathbf{r}) \, d^2 r$$

$$= \frac{1}{M} \int_{body} \mathbf{r} \, \rho(\mathbf{r}) \, d^3 r$$

*Ex1:*長 L 質量為 M 的均勻細棒質心位置

Ex.2 由線密度 λ 的均勻細金屬線彎成半徑 R 的半圓形之質心位置

C. 二項式

Binomial Theorem

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \ldots \qquad (x^2 < 1)$$

D. 微分方程-----以 考慮空氣阻力的自由落體

Ch.　Introduction and Vectors

Physics：study 1. composition of **matter** → Elementary Particles

2. **interaction** between particles → Fundamental Forces

Category of Physics：

Classical Physics	Modern Physics
Mechanics, Thermodynamics, Optics, and Electromagnetism	Relativity and Quantum Physics

◎ 1-1　Standards of Length, Mass, and Time

SI Units

Measurements → standards or Units

International System of Units (SI units)：

★ **Base units**：

Physical Quantity	Name	Symbol
Length	**meter**	**m**
Mass	**kilogram**	**kg**
Time	**second**	**s**
Temperature	**ampere**	**A**
Electric Current	**kelvin**	**K**
Amount of Substance	**mole**	**mol**
Luminous Intensity	candela	Cd

Derived units：combinations of base units

Ex: (1) The SI unit of force is called the newton (N), N = kg·m/s^2 ($F = ma$)

(2) The SI unit of energy or work is called the joule (J)

J = N·m=kg·m^2/s^2 (W=F·Δs)

★ **Table 1.4**：Some Prefixes for Powers of Ten

Factor	10^{15}	10^{12}	10^{9}	10^{6}	10^{3}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}
Prefix	peta	tera	giga	mega	kilo	centi	milli	micro	nano	pico	femto
Symbol	P	T	G	M	K	c	m	μ	n	p	f

◎ 1-2 Dimensional Analysis *(p8)*

—Check Units

We say the *dimension* of a distance is *length*, 【L】

Ex:

N = kg·m/s^2, the dimension of force 【*F*】= MLT^{-2}

An equation must be dimensionally consistent.

e.g. $x = at^2/2 \Rightarrow$ L = (LT^{-2})(T^2) = L

$$T = 2\pi\sqrt{\frac{l}{g}} \rightarrow [T]=[l/g]^{1/2} \rightarrow T=[\frac{L}{L/T^2}]^{1/2}$$

p.9 Exercise 3 and Example 1.2

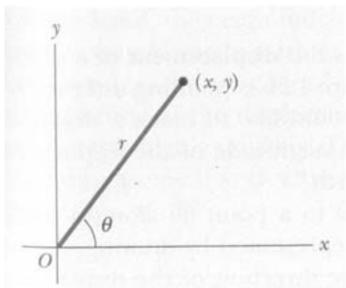
◎ 1-3 Reference Frames and Coordinate Systems

A **coordinates system** used to specify the position in space consists of

(1) A fixed reference point ***O***, called the origin.

(2) A set of axes.

*Fig*1.1 →



1. **Cartesian coordinate system** (or **rectangular coordinate system**) (**Fig. 1.1**)
   —*x*, *y*, and *z* axis, mutually perpendicular.
2. **Plane polar coordinates**：(*r*, *θ*)
   Transformation between Cartesian and plane polar coordinates：

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

or

$$\begin{cases} \tan\theta = y/x \\ r = \sqrt{x^2 + y^2} \end{cases}$$

◎ 1-4 Vectors and Scalars

A physical quantity：

scalar	mass, work, energy, power, ...
vector	position, displacement, velocity, force,........

scalar a : magnitude

Ex: vector $\vec{A}$: magnitude ($|\vec{A}|$ or A) and direction.

◎ 1-5 Some Properties of Vectors

1. Equality of two vectors：

$\vec{A}=\vec{B} \Leftrightarrow$ (1) $A=B$

(2) $\vec{A}, \vec{B}$ in the same direction

2. Multiplication of a vector by a Scalar

$\alpha\vec{A}$: ① if $\alpha>0$，then $|\alpha\vec{A}|=\alpha|\vec{A}|$, direction unchanged

② if $\alpha<0$，then $|\alpha\vec{A}|=-\alpha|\vec{A}|$, direction opposed

3. **Addition**：

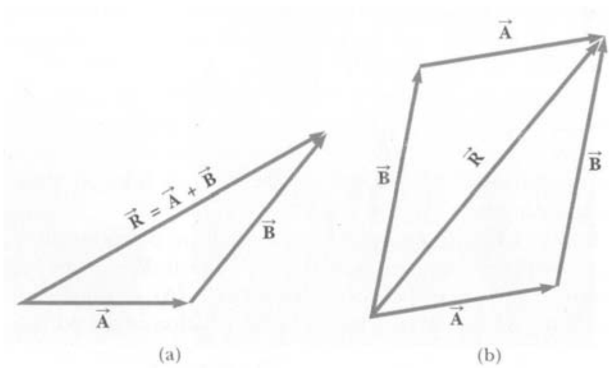


Fig. 1.2： $\vec{R}=\vec{A}+\vec{B}=\vec{B}+\vec{A}$ ← *Triangle method of addition*

4. **Subtraction of Vectors**

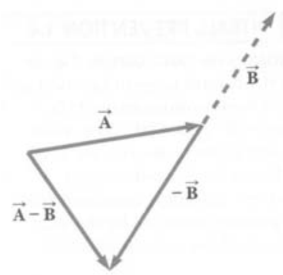


Fig. 1.3： $\vec{R}=\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

◎ 1-6 Components of a Vector and Unit Vectors

1. Use **unit vectors** $\hat{i}$, $\hat{j}$, $\hat{k}$ — lie along +*x*, +*y*, +*z* axis respectively

Unit vector ： The magnitude is 1

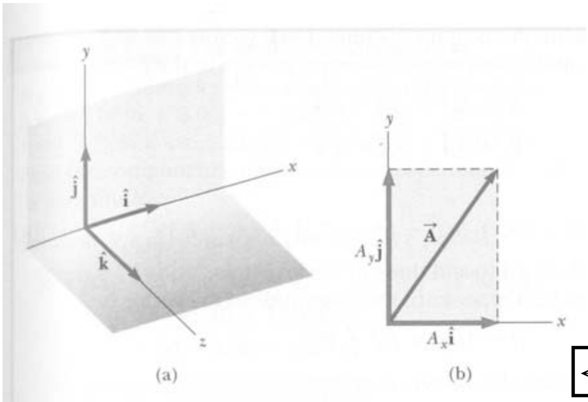
See **Fig. 1.4**

$\because \quad \vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$

and $\vec{A}_x = A_x\hat{i}$, $\vec{A}_y = A_y\hat{j}$, $\vec{A}_z = A_z\hat{k}$

$\therefore \quad \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ --------------- **[1.1]**

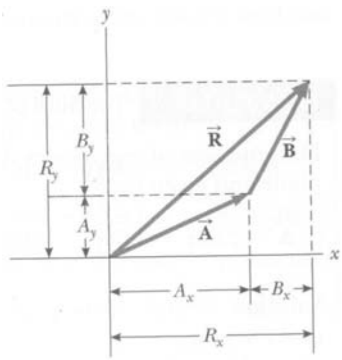
(A_x, A_y, A_z are called ***components*** of $\vec{A}$ along the *x*, *y*, *z* axis)



←Fig. 1.4

2. The magnitude of $\vec{A}$ is given by

*Fig*1.5→



$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ ------------ **[1.2]**

Given $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

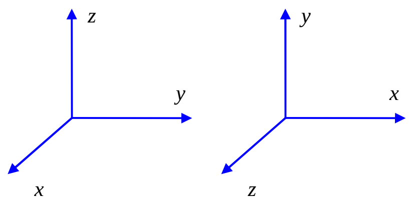
$\Rightarrow$ we obtain $\vec{R} = \vec{A} + \vec{B}$ **Fig. 1.5** (2 dim. case)

$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$ ------------ [1.3]

i.e. $R_x = A_x + B_x$, $R_y = A_y + B_y$, $R_z = A_z + B_z$ ------------- [1.4]

◎ 1-6 Scalar (dot) Product

$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A} = AB\cos\theta$



◎ 1-5 Vector (Cross) Product

$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$ vector

$\hat{i} \times \hat{i} = 0$ $\hat{j} \times \hat{i} = -\hat{k}$ $\hat{k} \times \hat{i} = 0$

$\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{j} = 0$ $\hat{k} \times \hat{j} = \hat{k}$

$\hat{i} \times \hat{k} = -\hat{j}$ $\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{k} = 0$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Ex 2.4：Find the scalar and vector products for $\vec{A} = 8\hat{i} + 2\hat{j} - 3\hat{k} = (8, 2, -3)$, and

$\vec{B} = 3\hat{i} - 6\hat{j} + 4\hat{k} = (3, -6, 4)$ 。

Ex 2.7：Find the scalar and vector products $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k} = (3, -2, 1)$,

$\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k} = (1, 4, , -2)$ 。

sol： $\vec{A} \times \vec{B} = 7\hat{j} + 14\hat{k}$

Electric Potential

◎ Potential

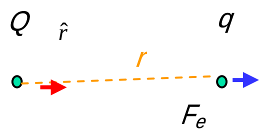
1. Electrostatic force is conservative

電學： electrostatic force

$\vec{F}_e = \frac{kq_1q_2}{r^2}\hat{r}$

for a point charge

$\vec{E}_e = \frac{kq_1}{r^2}\hat{r}$



力學： gravitational force:

$\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{r}$

$\vec{g} = \frac{Gm_1}{r^2}\hat{r}$

➢ $\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{r} = F(r)\hat{r}$ (Central force) $\vec{g} = \frac{Gm}{r^2}\hat{r}$ $\Rightarrow \vec{F}_g$: conservative force

➔ gravity potential energy ➔ $W = \int \vec{F}\cdot d\vec{s}$ Independent of the path

and only depend on the initial and final position

➢ electrostatic force： $\vec{F}_e = \frac{kq_1q_2}{r^2}\hat{r} = F(r)\hat{r}$ (Central force) ∴the same form as $\vec{F}_g$

$W = \int \vec{F}\cdot d\vec{s} = \int_{r_i}^{r_f} F(r)dr$ ∴ conservative force

➔ electric potential energy (電位能)

2. electric potential energy

$\Delta U_e = -W_{internal} = -\int \vec{F}_e \cdot d\vec{s}$ (∵ electrostatic force $\vec{F}_e = q\vec{E}$)

$= -\int q\vec{E}\cdot d\vec{s}$

$U_f - U_i = -\int_i^f q\vec{E}\cdot d\vec{s}$

① if point charge $\vec{F}_e = \frac{kq_1Q}{r^2}\hat{r}$ $U_f - U_i = -\int_{r_i}^{r_f} \frac{kqQ}{r^2}dr$

$= -kqQ(-\frac{1}{r})\Big|_{r_i}^{r_f} = \frac{kqQ}{r_f} - \frac{kqQ}{r_i}$

Assume $U(\infty) = 0$ $U_f - \underline{U_i(\infty)} = \frac{kqQ}{r_f} - 0$

∴ $U(r) = \frac{kqQ}{r}$ for $r \to \infty$ $U_\infty = 0$

The potential energy is the external work needed to bring the charge q from ∞ to the separation r

② a system of point charges

$$U=\frac{kq_1q_2}{r_{12}}+\frac{kq_1q_3}{r_{13}}+\frac{kq_2q_3}{r_{23}}+\cdots\cdots \quad =\sum\frac{kq_iq_j}{r_{ij}}$$

3. (electric) potential 電位 (只與空間有關)

<Note>

- 電位能 potential energy $U_{(r)}=\frac{kqQ}{r}$ a property of point charges (including the test chargeq) in space

  力和場(場只和空間有關)

- 電位 potential $\qquad \vec{F}=\frac{kqQ}{r^2}$

$$\Delta V\equiv\frac{\Delta U}{q}=\frac{\mathbf{1}}{q}\left(-\int q\vec{E}\cdot d\vec{s}\right)$$

$$=-\int\vec{E}\cdot d\vec{s}$$

Assume $V(\infty)=0$ for a point charge:

$$\triangle V=\ V_f-V_i=-\int\frac{kQ}{r^2}d\hat{r}=-(-\frac{kQ}{r_f}-\frac{kQ}{r_i})$$

$$=\frac{kQ}{r_f}-\frac{kQ}{r_i}$$

$$\therefore\quad V_f=\frac{kQ}{r_f}\Rightarrow V_{(r)}=\frac{kQ}{r}$$

<Note> 若∞電位不能設成 0 時，只能算電位差

4. SI Unit :

(1)U(電位能):J or $N\bullet m$

(2)V(電位): J/c or (N/c)m or V(伏特)

(3)E 電場: N/c or $\frac{V}{m}$

4.電場有向量，電位為純量

靜電力才是保守力(若非靜電力，則非保守力)

<Note>: 非保守力的判斷法

Ex: $\vec{F}=xy\hat{i}$ (非保守力) 保守力:力的函數只能有一個變數

5. $\mathbf{1}\ eV = \mathbf{1.602 \times 10^{-19}}\ J$ $\because$ $1\ e=1.6*10^{-19}\ C$

◎ Potential and potential energy in an uniform E

1. if $\vec{E} = -E\hat{j}$ $V_f - V_i = -\int \vec{E}\cdot d\vec{s} = -\int \vec{E}\cdot(dx\hat{i} + dy\hat{j}) = E\int_i^f dy = Ed$ (ref Fig 25.3)
2. Equipotential : electric field lines are perpendicular to the equipotentials and point from higher to lower potential
3. potential energy of point charges $U=qV$

   for point charge q at the distance r from the source charge Q

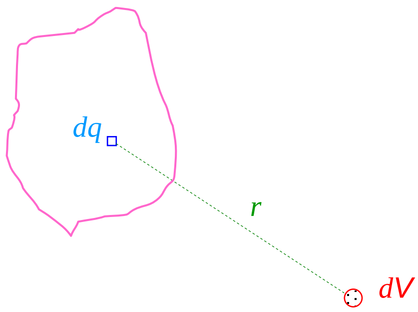
   $$U(r) = \frac{kqQ}{r} \qquad \text{for } U(\infty) = \mathbf{0}$$

◎ § Continuous charge distributions

1. 用庫侖定律算

If $V(\infty) = 0$ $V_P = \int dV$ (scalar 純量)

$$= \int \frac{kdq}{r}$$

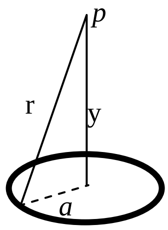


<Note> ∞長直導線在∞處的電位$\neq 0$ ($\because$有電荷)

It is not suitable for an infinite distribution of charge

→ ($V(\infty) \neq 0$)只能算電位差

◎ 2. $V_B - V_A = -\int \vec{E}\cdot d\vec{s}$ ←用 Gaussian Law



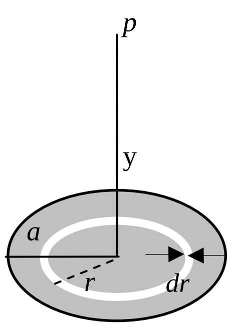
Ex: Find V_P at a point on the axis of a charge ring

<sol>

Set $V(\infty) = 0$

$$V_P = \int dV_P = \int \frac{kdg}{r} = \int \frac{kdq}{(a^2+y^2)^{1/2}} = \frac{k}{(a^2+y^2)^{1/2}}\int dq$$

$$= \frac{kQ}{(a^2+y^2)^{1/2}}$$



Ex 5 Find V_P at a point on the axis of a nonconducting charge

disk with radius a and surface charge density $\sigma\ {}^{c}\!/\!_{m^2}$?

<sol> 圓盤是由無線多個半徑不同的圓環所構成的

$$V_{環} = \frac{kQ}{(a^2+y^2)^{1/2}}$$

$$dV_{環} = \frac{kdq}{(r^2+y^2)^{1/2}} \qquad dq = \sigma 2\pi r dr$$

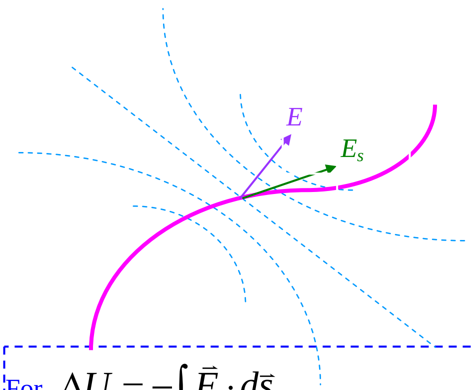
$$V_{disk} = \int dV_{環} = \int \frac{kdq}{(r^2+y^2)^{1/2}}$$

$$= k\sigma 2\pi \int_0^a \frac{r}{(r^2+y^2)^{1/2}} dr \quad = 2\sigma k\pi (r^2+y^2)^{1/2}\Big|_0^a \quad = 2\sigma k\pi \left[(a^2+y^2)^{1/2} - y\right]$$

If y>>a , $\frac{a}{y} \to 0$ $\quad V_P = 2\pi k\sigma[y(1+(\frac{a}{y})^2)^{1/2} - y] \quad = 2\pi k\sigma[y(1+\frac{1}{2}(\frac{a}{y})^2) - y]$

$$= 2\pi k\sigma \frac{a^2}{2y} \quad = \frac{k\pi a^2 \sigma}{y} \quad = \frac{kQ}{y} \qquad (\pi a^2 \sigma = \sigma A = Q)$$

另一種算法:先求電場, 由電場求電位



$$\Delta V = \int dV = -\int \vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s} = -(E_x dx + E_y dy + E_z dz)$$

$$\Rightarrow Ex = \frac{-\partial V}{\partial x} \quad , \quad Ey = \frac{-\partial V}{\partial y} \quad , \quad Ez = \frac{-\partial V}{\partial z}$$

$$\vec{E} = (-\frac{\partial V}{\partial x})\hat{i} + (-\frac{\partial V}{\partial y})\hat{j} + (-\frac{\partial V}{\partial z})\hat{k}$$

$$= -(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})V$$

For $\Delta U = -\int \vec{F} \cdot d\vec{s}$

(1) For 1 dim : $F = \frac{-\partial U}{\partial x}$

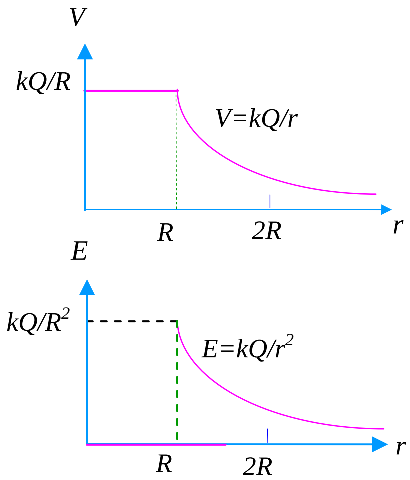
(2) For 2 dim : $F_x = \frac{-\partial U}{\partial x}$; $F_y = \frac{-\partial U}{\partial y}$

(3) For 3 dim : $F_x = \frac{-\partial U}{\partial x}$; $F_y = \frac{-\partial U}{\partial y}$; $F_z = \frac{-\partial U}{\partial z}$

$$\Rightarrow \vec{F} = -\vec{\nabla} U$$

Ex A Charged sphere shell , charge Q , R, find the $V_{(r)}$ for $^{(a)} r > R$ $^{(b)} r < R$ (the same as charge spherical conductor)

<sol> for r>R



$$1^0\ \Phi = \oint \vec{E}\cdot d\vec{A} = \frac{Qin}{\in_0}$$

$$E4\pi r^2 = \frac{Q}{\in_0}$$

$$\therefore \vec{E} = \frac{Q}{4\pi \in_0 r^2}\hat{r} = \frac{kQ}{r^2}\hat{r}$$

$$2^0\ \Delta V = \frac{\Delta U}{q} = \frac{1}{q}(-)\int q\vec{E}\cdot d\vec{s} = -\int \vec{E}\cdot d\vec{s}$$

$$V_{(r)} - V_{(R)} = -\int_R^r \underline{E(r)dr}$$

$$\downarrow$$

$$0$$

$$\therefore V_{(r)} = V_{(R)} = \frac{Q}{4\pi \in_0 R}$$

<Note> E-r , V-r 圖

Ex Uniform distribution charged sphere Q , R , find potential for $^{(a)}r > R$ $^{(b)}r < R$

<sol>

(a) outside , r>R

$$1^0\ E4\pi r^2 = \frac{Q}{\in_0} \qquad \therefore \mathrm{E} = \frac{\mathrm{Q}}{4\pi r^2 \in_0} \propto \frac{1}{r^2}$$

$$2^0\quad \Delta \mathrm{V} = -\int \vec{E}\cdot d\vec{s} \qquad \mathrm{V}_{(r)} - \mathrm{V}_{(R)} = -\int_\infty^r \frac{Q}{4\pi \in_0 r^2}dr$$

$$= -\frac{Q}{4\pi \in_0}\int_\infty^r \frac{1}{r^2}dr$$

$$= -\frac{\mathrm{Q}}{4\pi \in_0}\Big[-\frac{1}{r}\Big]_\infty^r$$

$$= \frac{\mathrm{Q}}{4\pi \in_0 r}$$

Let r=R

$$\mathrm{V}_{(r)} = \mathrm{V}_{(R)} \qquad \therefore \mathrm{V}_{(R)} = \frac{Q}{4\pi \in_0 R}$$

(b) inside r<R

$$1^0 \oint \vec{E} \cdot d\vec{A} = \frac{Qin}{\in_{\mathbf{0}}}$$

$$\mathrm{E}4\pi r^2 = \frac{Qin}{\in_0} = \frac{Qr^3 / R^3}{\in_0}$$

$$\therefore \mathrm{E} = \frac{\mathrm{Qr}}{4\pi \in_0 R^3} \propto r$$

$$2^0 \Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$\mathrm{V}_{(\mathrm{r})} - \mathrm{V}_{(\mathrm{R})} = -\int_R^r \frac{Qr}{4\pi \in_0 R^3} dr$$

$$\mathrm{V}_{(\mathrm{r})} - \frac{Q}{4\pi \in_0 R} = -\frac{Q}{4\pi \in_0 R^3} (\frac{1}{2}(r^2 - R^2))$$

$$\mathrm{V}_{(\mathrm{r})} = \frac{Q}{4\pi \in_0 R}(1 - \frac{1}{2}\frac{r^2}{R^2} + \frac{1}{2})$$

$$= \frac{Q}{4\pi \in_0 R}(3 - \frac{r^2}{R^2})$$